

1. p : It is raining q : It is pleasant

Then compound statement “It is not raining still it is pleasant” is _____

Sol. $\sim p \wedge q$

2. If A is a square matrix such that $A^2 = I$ then A^{-1} is equal to _____

Sol. $A^2 = I$

$$\therefore A^2 A^{-1} = I A^{-1}$$

$$\therefore A = A^{-1}$$

3. The general solution of the equation $7 \cos^2 \theta + 3 \sin^2 \theta = 4$ is

Sol. $7 \cos^2 \theta + 3 \sin^2 \theta = 4$

$$7 \cos^2 \theta + 3(1 - \cos^2 \theta) = 4$$

$$7 \cos^2 \theta + 3 - 3 \cos^2 \theta = 4$$

$$4 \cos^2 \theta + 3 = 4$$

$$4 \cos^2 \theta + 3 = 1 + 3$$

$$4 \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \pm \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = -\frac{1}{2}$$

$$\cos \theta = \cos \frac{\pi}{3} \quad \cos \theta = \cos \frac{2\pi}{3}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{3} \quad \theta = 2n\pi \pm \frac{2\pi}{3}$$

4. The solution of $(2 \cos x - 1)(3 + 2 \cos x) = 0$ in the interval $0 \leq x \leq 2\pi$ is

Sol. We have $(2 \cos x - 1)(3 + 2 \cos x) = 0$

$$2 \cos x - 1 = 0 \quad \text{or} \quad 3 + 2 \cos x = 0$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -\frac{3}{2}$$

$\cos x = -\frac{3}{2}$ which is not possible ($\because -1 \leq \cos x \leq 1$)

$$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$$

5. If one of the lines $2x^2 - xy - 15y^2 = 0$ is perpendicular to line $kx + y = 0$ then $k =$ _____

Sol. 3

6. If the angle between the two lines represented by $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$ is $\tan^{-1} m$, then $m = \underline{\hspace{2cm}}$

Sol. 1/5

7. The equation $9x^2 + 12xy + 4y^2 - 24x - 16y + 7 = 0$ represents a pair of _____ lines.

Sol. parallel

8. (1, -2) is the midpoint of chord AB of circle $x^2 + y^2 = 9$. Equation of AB is

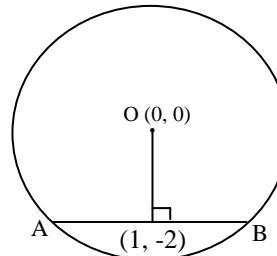
Sol. Slope of perpendicular from AB = $\frac{-2}{1}$

$$\therefore \text{slope of AB} = \frac{1}{2}$$

$$\text{Eq}^n \text{ of AB is } \frac{y+2}{x-1} = \frac{1}{2}$$

$$\therefore x - 1 = 2y + 4$$

$$\therefore x - 2y = 5$$



9. The lines $2x + 3y + 4 = 0$ and $3x - y - 5 = 0$ are diameters of a circle of area 154 sq. units, then the equation of the circle is

Sol. Centre of intersection of $2x + 3y + 4 = 0$ and $3x - y - 5 = 0$ which is (1, -2).

Area is 154 sq. units.

\therefore so $r = 7$.

$$\text{Eq}^n (x - 1)^2 + (y + 2)^2 = 49$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 49$$

$$x^2 + y^2 - 2x + 4y - 44 = 0$$

10. The eccentricity of an ellipse whose latus rectum is half of its minor axis is

Sol. $\frac{2b^2}{a} = b \quad \therefore 2b = a$

$\therefore b/a = 1/2$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

11. The acute angle between the lines whose direction cosines are proportional to $(2, 3, -6)$ and $(3, -4, 5)$ is _____

Sol. The actual d.c.'s of the lines are

$$\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7} \right) \text{ and } \left(\frac{3}{\sqrt{50}}, \frac{-4}{\sqrt{50}}, \frac{5}{\sqrt{50}} \right)$$

If θ is the angle between them, then

$$\begin{aligned} \cos \theta &= \pm \frac{(2)(3) + (3)(-4) + (-6)(5)}{7\sqrt{50}} = \pm \frac{6 - 12 - 30}{7\sqrt{50}} \\ &= \pm \frac{-36}{7\sqrt{50}} = \pm \frac{36}{7\sqrt{50}} \end{aligned}$$

12. The distance between the planes $\bar{r} \cdot (2\hat{i} - \hat{j} + 3\hat{k}) = 4$ and $\bar{r} \cdot (6\hat{i} - 3\hat{j} + 9\hat{k}) + 13 = 0$ is _____

Sol.
$$\frac{4 - \frac{-13}{3}}{\sqrt{(2)^2 + (-1)^2 + (3)^2}} = \frac{\frac{25}{3}}{\sqrt{14}} = \frac{25}{3\sqrt{14}}$$

13. The equation of the plane passing through $(1, 1, 0)$, $(-1, 3, 4)$ and perpendicular to the plane $x + y - 2z + 3 = 0$ is _____

Sol. The equation of the plane containing $(1, 1, 0)$ is

$$a(x - 1) + b(y - 1) + c(z - 0) = 0 \quad \text{It also contains } (-1, 3, 4)$$

$$a(-1 - 1) + b(3 - 1) + c(4 - 0) = 0$$

$$\therefore -2a + 2b + 4c = 0$$

$$\therefore -a + b + 2c = 0$$

Also the plane is perpendicular to $x + y - 2z + 3 = 0$

\therefore By condition of perpendicularity, $a + b - 2c = 0$

$$\therefore \frac{a}{-4} = \frac{b}{0} = \frac{c}{-2} \quad \therefore a = 2, b = 0, c = 1$$

$$\therefore \text{Required equation of plane } 2(x - 1) + 0(y - 1) + 1(z - 0) = 0$$

$$2x - 2 + 0 + z = 0 \quad \therefore 2x + z = 2$$

14. Let $z = 2x + 2y$, subject to restriction $x + y \leq 1$, $x \geq 0$, $y \geq 0$. The minimum value of z under given constraint is _____

Sol. The minimum value of z at 0 is 0.

15. Let $z = 15x + 10y$ subject to constraints $3x + 2y \leq 12$, $2x + 3y \leq 15$, $x \geq 0, y \geq 0$. The maximum value of z is _____

Sol. OABC is the feasible region

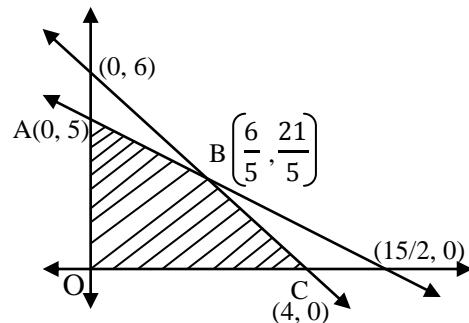
$$Z(0) = 15(0) + 10(0) = 0$$

$$Z(A) = 15(0) + 10(5) = 50$$

$$Z(B) = 15(6/5) + 10(21/5) = 18 + 42 = 60$$

$$Z(C) = 15(4) + 10(0) = 60$$

∴ Maximum value of Z is 60.



16. The differential equation of the family of circles touching y-axis at the origin is

Sol. As required circle touches y-axis at the origin.

∴ Let Centre of the circle is $d(a, 0)$ and radius is a

∴ Equation of circle will be,

$$(x-a)^2 + (y-0)^2 = a^2$$

$$\Rightarrow x^2 - 2ax + a^2 + y^2 = a^2$$

$$\Rightarrow x^2 + y^2 - 2ax = 0 \quad \dots(i)$$

By differentiating above equation w.r.t. x , we get

$$2x + 2y \frac{dy}{dx} - 2a = 0$$

$$\therefore 2a = 2x + 2y \frac{dy}{dx} \quad \dots(ii)$$

From (i) and (ii),

$$x^2 + y^2 - \left(2x + 2y \frac{dy}{dx} \right)x = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2xy \frac{dy}{dx} = 0$$

$$\Rightarrow x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

17. If Rolle's theorem for $f(x) = e^x (\sin x - \cos x)$ is verified on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ then the value of c is

Sol. Given, $f(x) = e^x (\sin x - \cos x)$

$$\therefore f'(x) = e^x [\cos x + \sin x] + [\sin x - \cos x]e^x$$

$$\therefore f'(x) = 2e^x \sin x$$

To verify Rolle's Theorem.

$$f'(c) = 0$$

$$2e^c \sin c = 0$$

$$\Rightarrow \sin c = 0 \quad \therefore c = \pi$$

18. For what value of k, the function defined by $f(x) = \begin{cases} \frac{\log(1+2x) \sin x^\circ}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$ for $x \neq 0$
is continuous at $x = 0$?

Sol. As given, $f(x) = \begin{cases} \frac{\log(1+2x) \sin x^\circ}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$

Is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0} \frac{\log(1+2x) \cdot \sin x^\circ}{x^2} = k$$

$$\therefore \lim_{x \rightarrow 0} \frac{2 \cdot \log(1+2x)}{2x} \cdot \lim_{x \rightarrow 0} \frac{\sin\left(x \times \frac{\pi}{180}\right)}{\left(x \times \frac{\pi}{180}\right)} \times \frac{\pi}{180} = k$$

$$\Rightarrow 2 \times \frac{\pi}{180} = k$$

$$\Rightarrow \frac{\pi}{90} = k$$

19. The degree and order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ respectively are

Sol. Degree $\Rightarrow 3$ Order $\Rightarrow 2$

20. $\int \frac{f(x)}{\log(\sin x)} dx = \log[\log \sin x] + C$, then $f(x) =$

Sol. Given $\int \frac{f(x)}{\log(\sin x)} dx = \log [\log \sin x] + C$

By using anti differentiation method, we will get

$$\begin{aligned} & \frac{d}{dx} (\log [\log \sin x] + C) \\ &= \frac{1}{\log(\sin x)} \cdot \frac{1}{\sin x} \cdot \cos x \\ &= \frac{\cot x}{\log(\sin x)} \quad \therefore f(x) = \cot x \end{aligned}$$

21. The particular solution of the differential equation $y(1 + \log x) \frac{dx}{dy} - x \log x = 0$ when $x = e, y = e^2$ is

Sol. $y(1 + \log x) \frac{dx}{dy} - x \log x = 0$

$$\Rightarrow \left(\frac{1 + \log x}{x \log x} \right) dx = \frac{dy}{y}$$

Integrating on both side

$$\int \left(\frac{1 + \log x}{x \log x} \right) dx = \int \frac{dy}{y}$$

$$\log(x \log x) = \log y + \log C$$

$$\Rightarrow \log(x \log x) = \log(y \cdot c)$$

$$\therefore x \log x = y \cdot c \quad \dots(i)$$

$$\text{As } x = e, \quad y = e^2$$

$$\therefore e = e^2 \cdot c \quad \therefore c = \frac{1}{e}$$

$$\text{Putting } c = \frac{1}{e} \text{ in eq (i) we get}$$

$$x \log x = \frac{y}{e} \quad \Rightarrow \quad y = ex \log x$$

22. If r.v. $X \sim B\left(n=5, P=\frac{1}{3}\right)$ then $P(2 < X < 4) = \underline{\hspace{2cm}}$

Sol. Given $n = 5, p = \frac{1}{3}$

$$\therefore q = \frac{2}{3}$$

$$p(2 < x < 4) = p(x = 3)$$

$$= {}^5C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2$$

$$= \frac{5 \times 4}{2} \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$$

23. Let X has p.m.f.

$$\begin{aligned} P(x) &= kx^2, x = 1, 2, 3, 4 \\ &= 0, \quad \text{otherwise} \end{aligned}$$

Sol. $\sum P(x) = 1$

$$P(1) + P(2) + P(3) + P(4) = 1$$

$$30k = 1$$

$$\therefore K = \frac{1}{30}$$

24. $F(x)$ associated with the following p.d.f. $f(x)$

$$f(x) = \begin{cases} 3(1 - 2x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

then value of $P(1/4 < x < 1/3)$ by using p.d.f.

Sol. $P\left(\frac{1}{4} < x < \frac{1}{3}\right) = \int_{1/4}^{1/3} f(x) dx$

$$\begin{aligned} &= \int_{1/4}^{1/3} 3(1 - 2x^2) dx \\ &= [3 - 2x^3]_{1/4}^{1/3} \end{aligned}$$

$$= \frac{179}{864} = 0.2071$$

25. If a fair coin is tossed 8 times then probability that it shows head more often than tail is

Sol. $P(\text{more heads than tails})$

$$\begin{aligned} &= P(x = 5 \text{ or } 6 \text{ or } 7 \text{ or } 8) \\ &= P(x = 5) + P(6) + P(7) + P(8) \\ &= \frac{1}{2^8} \left[8C_3 + 8C_2 + 8C_1 + 8C_0 \right] \\ &= \frac{93}{256} \end{aligned}$$

26. The degree of the differential equation satisfying $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y)$ is

Sol. $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y)$

Diff w.r.t 'x'

$$\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}} \cdot \frac{a\sqrt{1+x^2-x}}{a\sqrt{1+y^2+y}}$$

But from the given relation

$$\begin{aligned} \left[\sqrt{1+x^2} + \sqrt{1+y^2} \right] &= \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\ &= a(x-y) \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\ &= 1+x^2 - 1-y^2 = a(x-y) \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\ &= x+y = a \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\ &= y + a\sqrt{1+y^2} = a\sqrt{1+x^2} - x = \frac{a\sqrt{1+x^2-x}}{a\sqrt{1+y^2+y}} = 1 \\ \therefore \frac{dy}{dx} &= \sqrt{\frac{1+y^2}{1+x^2}} \end{aligned}$$

Clearly it is a differential equation of first order and first degree.

27. The area common to circle $x^2 + y^2 = 36$ and line $x + y = 6$ is _____

Sol. The area bounded by the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ is given by

$$\begin{aligned} &\frac{a^2}{4} (\pi - 2) \text{ sq. units} \\ &= \frac{36}{4} (\pi - 2) \text{ sq. units} \\ &= 9(\pi - 2) \text{ sq. units} \end{aligned}$$

28. The area of the region common to $y^2 = 4x$ and $x^2 = 4y$ is

Sol. By area of the region bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16ab}{3}$

29. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}} dx$ is equal to

Sol. $\int \frac{(x^2-1) dx}{(x^2+2x+1)x\sqrt{x+\frac{1}{x}+1}}$ is equal to

$$\int \frac{\left(1-\frac{1}{x^2}\right) dx}{\left[1-\frac{1}{x}+2\right]\sqrt{x+\frac{1}{x}+1}}$$

$$\int \frac{2z dz}{(z^2+1)z}$$

$$\text{Putting } x + 1/x + 1 = z^2$$

$$= \left(1 - \frac{1}{x^2}\right) dx = 2z dz$$

$$= 2 \int \frac{dz}{1+z^2}$$

$$= 2 \tan^{-1} z + C$$

$$= 2 \tan^{-1} \left(\sqrt{\frac{x^2+x+1}{x}} \right) + C$$

30. The area of the triangle formed by the positive x - axis, the tangent and the normal to the circle $x^2 + y^2 = g$ at $(2, \sqrt{5})$ is

Sol. $x^2 + y^2 = g$

$$2x + 2y \frac{dy}{dx} = 0$$

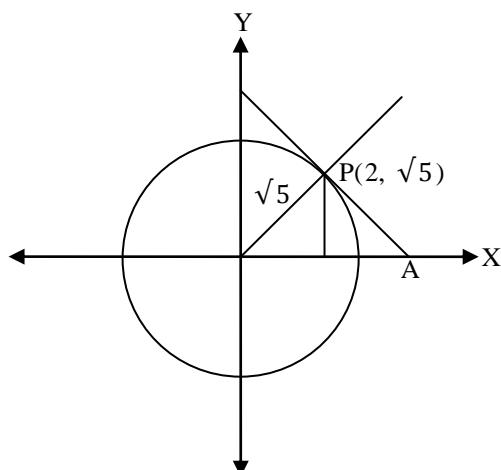
$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{5}}$$

The equation of tangent is

$$(y - \sqrt{5}) = \frac{-2}{\sqrt{5}}(x - 2)$$

The equation of normal is

$$(y - \sqrt{5}) = \frac{\sqrt{5}}{2}(x - 2)$$



The tangent and normal intersect at x-axis at the points $A(9/2, 0)$ and $O(0, 0)$ respectively.

$$\therefore A(\Delta OAP) = \frac{1}{2} \times \frac{9}{2} \times \sqrt{5} = \frac{9\sqrt{5}}{4}$$

31. $\int_0^{\pi/4} x \cdot \sec^2 x \, dx =$

Sol.
$$\begin{array}{ccc} x & \xrightarrow{+} & 1 \\ \sec^2 x & & \tan x \xrightarrow{-} 0 \\ & & \ln \sec x \end{array}$$

$$I = (x \tan x - \ln \sec x) \Big|_0^{\pi/4} = \frac{\pi}{4}(1) - \ln \sqrt{2}$$

$$I = \frac{\pi}{4} - \log \sqrt{2}$$

32. Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ then the value of $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ is

Sol. $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A|$
 $|A| = 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$
 $= -13 + 6 + 6$
 $= -1$
 $|A| = -1$

33. If $\int_0^k \frac{dx}{2 + 18x^2} = \frac{\pi}{24}$, then the value of k is

Sol. $\frac{1}{2} \int_0^k \frac{dx}{1 + 9x^2} = \frac{\pi}{24}$

$$\int_0^k \frac{dx}{1 + (3x)^2} = \frac{\pi}{12}$$

$$\left(\frac{1}{3} \tan^{-1}(3x) \right)_0^k = \frac{\pi}{12}$$

$$\tan^{-1} 3k = \frac{\pi}{4}$$

$$3k = 1$$

$$k = \frac{1}{3}$$

34. $\int \frac{1}{\sin x \cdot \cos^2 x} dx =$

Sol. $I = \int \frac{1}{\sin x \cdot \cos^2 x}$
 $I = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos^2 x} dx$
 $I = \int \frac{\sin^2 x}{\sin x \cdot \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \cos^2 x} dx$
 $I = \int \sec x \tan x dx + \int \csc x \sec x dx$
 $I = \sec x + \ln |\csc x - \cot x| + c$

35. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{y-k}{2} = z$ intersect the value of k is

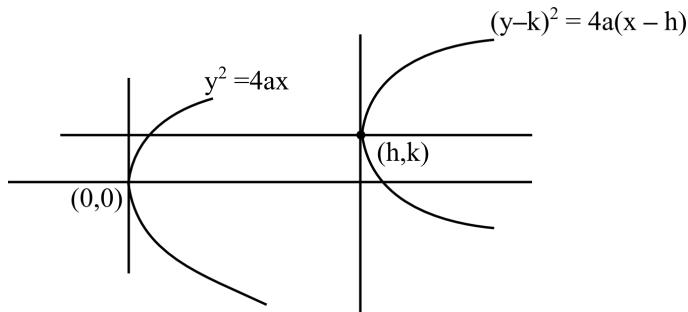
Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$
 $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1}$
 $\begin{bmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} = 0$
 $\begin{bmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} = 0$
 $2(-5) - (k+1)(-2) - 1(1) = 0$
 $-10 + 2k + 2 - 1 = 0$
 $2k = 9$
 $k = \frac{9}{2}$

36. $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 179^\circ =$

Sol. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos 179^\circ$
 $\therefore \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots 0 \dots \cos 179^\circ$
 $= 0$

37. The order of the differential equation of all parabolas, whose latus rectum is $4a$ and axis parallel to the x -axis, is

Sol.



$$\therefore \text{Equation of parabola } (y - k)^2 = 4a(x - h)$$

have two arbitrary constants h and k

$$\therefore \text{Order} = 2$$

38. A die is thrown four times. The probability of getting perfect square in at least one throw is

Sol. $n = 4$

$$p = \frac{2}{6} = \frac{1}{3}$$

$$q = \frac{4}{6} = \frac{2}{3}$$

X = Number on die is perfect square.

$$P(X = 0) = {}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{16}{81} = \frac{65}{81}$$

39. If A, B, C are the angles of $\triangle ABC$ then $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A =$

Sol. $A + B + C = \pi$

$$A + B = \pi - C$$

$$\cot(A + B) = \cot(\pi - C)$$

$$\frac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$$

$$\cot A \cot B - 1 = -\cot A \cot C - \cot B \cot C$$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

40. The sum of the first 10 terms of the series $9 + 99 + 999 + \dots$ is

Sol. $S_n = (10 - 1 + 100 - 1 + 1000 - 1 + \dots)$

$$S_n = (10 + 100 + 1000 + \dots) - (1 + 1 + 1\dots)$$

$$S_n = 10 \left(\frac{10^n - 1}{10 - 9} \right) - n$$

$$S_{10} = 10 \left(\frac{10^{10} - 1}{9} \right) - 10$$

$$S_{10} = 10 \left(\frac{10^{10} - 1}{9} - 1 \right)$$

$$S_{10} = 10 \left(\frac{10^{10} - 1 - 9}{9} \right)$$

$$S_{10} = 10 \left(\frac{10^{10} - 10}{9} \right)$$

$$S_{10} = \frac{100}{9} (10^9 - 1)$$

41. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is :

Sol. $L : \frac{x+y}{a} = \frac{y-3}{b} = \frac{z-1}{c} \quad \dots(1)$

$$L \parallel x + 2y - z - 5 = 0 \Rightarrow a + 2b - c = 0 \quad \dots(2)$$

$$(1) \text{ intersects } \frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$$

$$\Rightarrow \begin{vmatrix} 3 & 0 & 1 \\ a & b & c \\ -3 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 3(-b - 2c) + 1(2a + 3b) = 0$$

$$\Rightarrow 2a + 0b - 6c = 0$$

$$\Rightarrow a + 0b - 3c = 0 \quad \dots(3)$$

$$a + 2b - c = 0 \quad \dots(2)$$

$$\frac{a}{6} = \frac{b}{-2} = \frac{c}{2}$$

$$\frac{a}{3} = \frac{b}{-1} = \frac{c}{1}$$

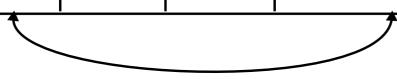
$$\Rightarrow L \text{ is: } \frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

42. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \vee q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair (\oplus, \odot) is :

Sol. $\oplus, \odot \in \{\wedge, \vee\}$

$$(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$$

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\sim p \wedge q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	F	T	T	F
F	F	T	F	F	T	F	F



43. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

Sol. Circle : $(x - 3)^2 + (y - 0)^2 = 3^2$

Centre : $(3, 0)$, radius : 3

Parabola : $y^2 = 4 \cdot 1 \cdot x$

tangent to parabola : $y = mx + \frac{1}{m}$

$$\Rightarrow m^2x - my + 1 = 0 \quad \dots(1)$$

If it is tangent of circle, then perpendicular from $(3, 0)$ on (1) = radius

$$\Rightarrow \left| \frac{3m^2 + 1}{\sqrt{m^2 + m^4}} \right| = 3$$

$$\Rightarrow 9m^4 + 1 + 6m^2 = 9m^4 + 9m^2$$

$$m^2 = \frac{4}{3} \Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

$$m = \frac{1}{\sqrt{3}} \text{ in (1)} \Rightarrow \frac{1}{3}x - \frac{1}{\sqrt{3}} \cdot y + 1 = 0$$

$$x - \sqrt{3}y + 3 = 0$$

$$\Rightarrow \sqrt{3}y = x + 3$$

Now take $m = -\frac{1}{\sqrt{3}}$

$$\frac{1}{3}x + \frac{1}{\sqrt{3}}y + 1 = 0$$

$$\Rightarrow x + \sqrt{3}y + 3 = 0$$

44. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and \vec{c} be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

Sol. $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$,

$$\text{let } \vec{c} = xi\hat{i} + y\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{c} + \vec{b} = 0$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ x & y & z \end{vmatrix} + (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\Rightarrow i(-2) - j(2) + k(y + x) + (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\Rightarrow 1 - 2 = 0 \Rightarrow 2 = 1$$

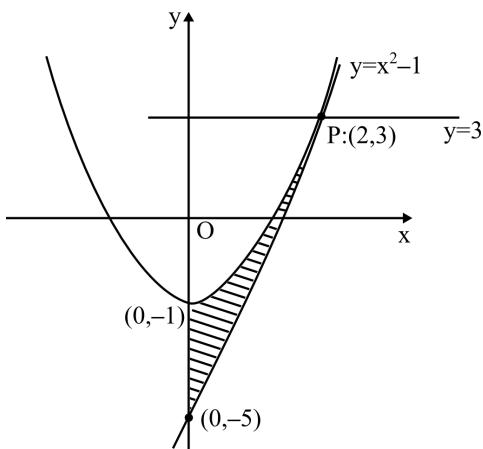
$$\left. \begin{array}{l} y + x = -1 \\ \vec{a} \cdot \vec{c} = 4 \Rightarrow x - y = 4 \end{array} \right\} \dots(1) \quad \left. \begin{array}{l} \dots(1) \\ \dots(2) \end{array} \right\} \Rightarrow x = \frac{3}{2}, y = -\frac{5}{2}$$

$$|\vec{c}|^2 = x^2 + y^2 + z^2 = \frac{9}{4} + \frac{25}{4} + 1 = \frac{38}{4} = \frac{19}{2}$$

...(1)

45. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y -axis is:

Sol.



$$y' = 2x$$

$$y = x^2 - 1$$

$$y'|_{2,3} = 4$$

$$x = \sqrt{1+y}$$

$$\text{tangent: } y - 3 = 4(x - 2)$$

$$y = 4x - 5$$

$$\text{Area required: } \frac{1}{2} \cdot 8 \cdot 2 - \int_{-1}^3 \sqrt{1+y} dy$$

$$= 8 - \frac{2}{3} \left[(1+y)^{3/2} \right]_{-1}^3$$

$$= 8 - \frac{2}{3} \cdot [8] = 8 \cdot \frac{1}{3} = \frac{8}{3}$$

46. The value of $\int_0^\pi |\cos x|^3 dx$ is :

$$\begin{aligned}
 \text{Sol. } & \int_0^\pi |\cos x|^3 dx = 2 \cdot \int_0^{\pi/2} \cos^3 x dx \\
 &= 2 \cdot \int_0^{\pi/2} \cos x \cdot \cos^2 x dx \\
 &= 2 \cdot \int_0^{\pi/2} (1 - \sin^2 x) \cdot \cos x dx \\
 &= 2 [\sin x]_0^{\pi/2} - 2 \left[\frac{\sin^3 x}{3} \right]_0^{\pi/2} \\
 &= 2 - 2 \cdot \frac{1}{3} = \frac{4}{3}
 \end{aligned}$$

47. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be:

$$\begin{aligned}
 \text{Sol. } & a, b, c : \text{distinct no. in G.P.} \\
 &= a, ar, ar^2 : r \neq 1, (r \text{ is common ratio}) \\
 & a + b + c = xb \\
 & \Rightarrow a + ar + ar^2 = x \cdot ar \\
 & \Rightarrow a \cdot (r^2 - rx + r + 1) = 0 \\
 & \Rightarrow a \cdot (r^2 - r(x-1) + 1) = 0 \\
 & a \neq 0, r \in R - \{1\} \\
 & D \geq 0 \Rightarrow (x-1)^2 - 4 \geq 0 \\
 & (x-1-2)(x-1+2) \geq 0 \\
 & (x+1)(x-3) \geq 0 \\
 & x \in (-\infty, -1] \cup [3, \infty) \\
 & \text{When } r = 1 \Rightarrow 1 - x + 1 + 1 = 0 \\
 & x = 3 \\
 & \text{as } r \neq 1 \Rightarrow x \neq 3 \\
 & \Rightarrow x \in (-\infty, -1] \cup (3, \infty) \\
 & \text{i.e. } x \neq 2
 \end{aligned}$$

48. If $\cos^{-1} \left(\frac{2}{3x} \right) + \cos^{-1} \left(\frac{3}{4x} \right) = \frac{\pi}{2}$ ($x > \frac{3}{4}$), then x is equal to :

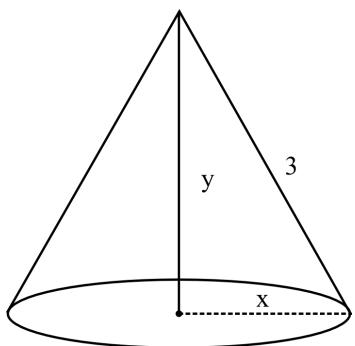
$$\begin{aligned} \text{Sol. } & \cos^{-1} \left(\frac{2}{3x} \right) + \cos^{-1} \left(\frac{3}{4x} \right) = \frac{\pi}{2}, x > \frac{3}{4} \\ & \Rightarrow \cos^{-1} \left(\frac{2}{3x} \right) = \frac{\pi}{2} - \cos^{-1} \left(\frac{3}{4x} \right) \\ & \Rightarrow \cos^{-1} \left(\frac{2}{3x} \right) = \sin^{-1} \left(\frac{3}{4x} \right) \\ & \Rightarrow \frac{2}{3x} = \sqrt{1 - \frac{9}{16x^2}} \\ & \Rightarrow \frac{4}{9x^2} = \frac{16x^2 - 9}{16x^2} \\ & \Rightarrow 64 = 144x^2 - 81 \\ & 144x^2 = 145 \\ & x^2 = \frac{145}{144} \Rightarrow x = \frac{\sqrt{145}}{12} \end{aligned}$$

49. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

$$\begin{aligned} \text{Sol. } & e > 2 \Rightarrow \sqrt{1 + \frac{b^2}{a^2}} > 2 \\ & \Rightarrow \sqrt{1 + \tan^2 \theta} > 2 \Rightarrow \tan^2 \theta > 3 \\ & \Rightarrow \theta \in (60^\circ, 90^\circ) \\ & \text{length of latus rectum : } \frac{2b^2}{a} \\ & = 2 \frac{\sin^2 \theta}{\cos \theta} = 2 \tan \theta \cdot \sin \theta = f(\theta) \\ & f'(\theta) > 0 \quad \forall \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2} \right) \\ & \Rightarrow \text{latus rectum} \in (3, \infty) \end{aligned}$$

50. The maximum volume (in cu. m) of the right circular cone having slant height 3m is:

Sol. $x^2 + y^2 = 9$



$$V = \frac{1}{3} \cdot \pi \cdot x^2 \cdot y = \frac{1}{3}\pi \cdot y(9 - y^2)$$

$$V = \frac{\pi}{3}(9y - y^3)$$

$$\frac{dv}{dy} = \frac{\pi}{3}(9 - 3y^2) = 0$$

$$y^2 = 3 \Rightarrow y = \pm\sqrt{3}$$

$$\begin{aligned} V|_{\max} &= \frac{\pi}{3} \cdot \sqrt{3}(9 - 3) \\ &= \frac{\pi}{3} \cdot \sqrt{3} \cdot 6 = 2\sqrt{3} \cdot \pi \end{aligned}$$